35.468 Computer Communication Networks **November 6, 2000**

Homework Assignment #5 **Solutions**

1. Consider a transmission link with fixed link capacity $C = 1.5\ Mb/s$, an infinite buffer, and a Poisson packet arrival process with rate $\lambda = 1000\ p/s$. Assume that the packet length distribution is exponential with mean $L = 1000\ b/p$.

   (a) Compute the mean number of packets in the system (including in the transmitter). Compute the mean delay for a packet.

   Solution:
   Using the "Single Link Example" from the notes, we have

   $$\mu = \frac{C}{L} = \frac{1.5\ Mb/s}{1000\ b/p} = 1500\ p/s$$

   So using our formulas for an M/M/1 queue, the link utilization is

   $$\rho = \frac{\lambda}{\mu} = 0.67$$

   The mean number of packets in the system (including in the transmitter) is

   $$E(n) = \frac{\rho}{1-\rho} = 2\ packets$$

   and the mean delay for a packet is

   $$E(T) = \frac{E(n)}{\lambda} \approx 2.0\ ms$$

   (b) Now assume that the arrival rate of packets has risen to $\lambda = 2000\ p/s$ so we double the transmission capacity to $C = 3\ Mb/s$. What happens to the mean number of packets in the system? What happens to the mean delay for a packet? Justify your answer computationally <u>and</u> intuitively.

   Solution:
   In this case we have

   $$\mu = \frac{C}{L} = \frac{3\ Mb/s}{1000\ b/p} = 3000\ p/s$$

   But the link utilization is still

   $$\rho = \frac{\lambda}{\mu} = 0.67$$

   So the mean number of packets in the system (including in the transmitter) is unchanged

   $$E(n) = \frac{\rho}{1-\rho} = 2.0\ packets$$

but the mean delay for a packet is

$$E(T) = \frac{E(n)}{\lambda} \approx 1\ ms$$

Half what it was in part (a).

Intuitively these results make sense because we've essentially speeded up the time scale of the system by a factor of two, so a packet spends half as long in the system as before, but the number in the system over time doesn't change. (Picture a video of the system in part (a) run at double speed.)

2. Consider a transmission link with a finite buffer modeled as an M/M/1/N queueing system, where N is the total number of buffer slots (including one in the server). Assume packet arrivals with rate $\lambda = 23\ p/s$ and fixed link capacity $C = 28,800\ b/s$ (T3 carrier rate). Approximate the packet length distribution by an exponential with mean $L = 1000\ b/p$. Compute the smallest buffer size, N, which would yield a blocking probability less than 10^{-4}.

<u>Solution</u>:

In this case

$$\mu = \frac{C}{L} = \frac{28800\ b/s}{1000\ b/p} = 28.8\ p/s$$

So,

$$\rho = \frac{\lambda}{\mu} \approx 0.8$$

For a buffer size of N, the blocking probability is

$$P_B = \frac{(1-\rho)\rho^N}{1-\rho^{N+1}}$$

We have $P_B > 10^{-4}$ for $N = 34$, while $P_B < 10^{-4}$ for $N = 35$, so 35 is the smallest buffer size which would yield a blocking probability less than 10^{-4}.

3. Consider a transmission link with fixed link capacity $C = 1.5\ Mb/s$, $\lambda = 750\ p/s$ and <u>mean</u> packet length $L = 1000\ b/p$. Using the Pollaczek-Khinchine (P-K) Formula to compute the delay $E(T)$ for the following three cases

   (i) All packets have the same length ($\sigma^2 = 0$),

   (ii) The packet length distribution is exponential,

   (iii) The variance of service time $\sigma^2 = 16 \times 10^{-6}$.

Solution:

Here

$$\mu = \frac{C}{L} = \frac{1.5\ Mb/s}{1000\ b/p} = 1500\ p/s$$

The Pollaczek-Khinchine (P-K) Formula states

$$E(n) = \left(\frac{\rho}{1-\rho}\right)\left[1 - \frac{\rho}{2}(1 - \mu^2\sigma^2)\right] \quad \rho < 1$$

where σ^2 is the variance of the service time distribution.

Applying Little's Formula we get the mean delay P-K Formula

$$E(T) = \frac{E(n)}{\lambda} = \left(\frac{1}{\mu-\lambda}\right)\left[1 - \frac{\rho}{2}(1 - \mu^2\sigma^2)\right] \quad \lambda < \mu$$

(i) All packets have the same length ($\sigma^2 = 0$).

With $\sigma^2 = 0$ in the P-K Formula, we get the M/D/1 formula

$$E(T) = \left(\frac{1}{\mu-\lambda}\right)\left(1 - \frac{\rho}{2}\right) \quad \lambda < \mu$$

$$E(T) = \left(\frac{1}{1500-750}\right)[1 - \frac{0.5}{2}]$$

Hence

$$E(T) = 1\ m/s$$

(ii) The packet length distribution is exponential.

Here we can plug in $\sigma^2 = \frac{1}{\mu^2}$ for the exponential distribution (from any probability book) or we can just use the M/M/1 delay formula

$$E(T) = \frac{1}{\mu-\lambda}$$

Hence,

$$E(T) = \frac{1}{1500-750} = 1.33\ ms$$

(iii) The variance of service time $\sigma^2 = 16 \times 10^{-6}$.

Plugging in $\sigma^2 = 16 \times 10^{-6}$ and the values given in the problem, we get

$$E(T) = \left(\frac{1}{\mu - \lambda}\right)\left[1 - \frac{\rho}{2}(1 - \mu^2\sigma^2)\right]$$

$$= \left(\frac{1}{1500 - 750}\right)\left[1 - \frac{0.5}{2}(1 - ((2.25 \times 10^6)(16 \times 10^{-6}))\right]$$

$$E(T) = 13\ ms$$

4. Consider the queueing network depicted below. All servers have a capacity of 5 packets/sec.

   (a) <u>Solution</u>

$$\begin{aligned}
\lambda_1 &= r_1 + \lambda_3 P_{31} = 3 + \frac{1}{2}\lambda_3 \\
\lambda_2 &= \lambda_1 \\
\lambda_3 &= r_3 + \lambda_1 P_{23} = 1 + \frac{1/4}{\lambda\ {}_2} \\
\lambda_4 &= \lambda_2 P_{24} + \lambda_3 P_{34} = \frac{3}{4}\lambda_2 + \frac{1}{2}\lambda_3
\end{aligned}$$

   Solve the above equations, we get

$$\lambda_1 = 4, \lambda_2 = 4, \lambda_3 = 2, \lambda_4 = 4$$

   (b) ¿From Jackson we can treat each queue as a separate M/M/1, so

$$\begin{aligned}
E(T_1) &= \frac{1}{5 - 4} = 1.0 \\
E(T_2) &= \frac{1}{5 - 4} = 1.0 \\
E(T_3) &= \frac{1}{5 - 2} = 0.33 \\
E(T_4) &= \frac{1}{5 - 4} = 1.0
\end{aligned}$$

   (c) The network-wide mean delay is given by

$$E(T) = \frac{1}{\gamma}\sum_{k=1}^{K}\frac{\lambda_k}{\mu_k - \lambda_k} = 3.17$$

since

$$\gamma = \sum_{k=1}^{K} r_k = 4$$

(d) Thus the mean delay for packet going Q3→Q1→Q2→Q4 is

$$E(T_3) + E(T_1) + E(T_2) + E(T_4) = 3.33$$

(e) (c) is the mean delay of different paths in the network. (d) is just the expectation delay of a particular path.