

Step	Router 2 $N_2(1), D_2(1)$	Router 3 $N_3(1), D_3(1)$	Router 4 $N_4(1), D_4(1)$	Router 5 $N_5(1), D_5(1)$	Router 6 $N_6(1), D_6(1)$
0	? ∞	? ∞	? ∞	? ∞	? ∞
1	1 2	? ∞	1 3	? ∞	? ∞
2	1 2	2 3	1 3	4 6	? ∞
3	1 2	2 3	1 3	4 6	3 5

Table 1: Distributed Bellman-Ford Algorithm for optimal routes when Node 1 was down and up now.

Step	Router 2 $N_2(1), D_2(1)$	Router 3 $N_3(1), D_3(1)$	Router 4 $N_4(1), D_4(1)$	Router 5 $N_5(1), D_5(1)$	Router 6 $N_6(1), D_6(1)$
Initial	1 2	2 3	1 3	4 6	3 5
1 Down	3 4		2 5		
(2- > 3,4) (4- > 2,5)		2 5	2 7	6 6	
(3- > 2,5,6) (4- > 2,5) (5- > 3,4,6)	3 6				3 7
(2- > 3,4) (6- > 3,5)		2 7	2 9	6 8	
(3- > 2,5,6) (4- > 2,5) (5- > 3,4,6)	3 8				3 9
(2- > 3,4) (6- > 3,5)		2 9	2 11	6 10	
(3- > 2,5,6) (4- > 2,5) (5- > 3,4,6)	3 10				3 11

Table 2: Shortest paths of the routing table after Node 1 goes down. Note that the process will continue until they reach some threshold value. After which the distance is set to be ∞ .

1. The assignment was to demonstrate each “show that” in the class overheads, *Network Layer Performance Modeling and Analysis, Part I: Essentials of Probability*.

Some useful formulas:

$$\begin{aligned} \sum_{x=0}^n \binom{n}{x} a^x b^{n-x} &= (a+b)^n \quad \text{for any } a, b \\ \sum_{x=0}^{\infty} a^x &= \frac{1}{1-a} \quad \text{for } a < 1 \\ \sum_{x=0}^{\infty} x a^x &= \frac{a}{(1-a)^2} \quad \text{for } a < 1 \\ \sum_{x=0}^{\infty} \frac{a^x}{x!} &= e^a \quad \text{for any } a \end{aligned}$$

2. From p. 19, for a binomial random variable show

$$\sum_{x=0}^n P_X(x) = 1$$

Solution:

$$\begin{aligned} \sum_{x=0}^n P_X(x) &= \sum_{x=0}^n \binom{n}{x} p^x (1-p)^{n-x} \\ &= [p + (1-p)]^n \\ &= 1 \end{aligned}$$

3. From p. 19, for a binomial random variable show

$$E(X) = np$$

Solution:

$$\begin{aligned} E(X) = \sum_{x=0}^n x P_X(x) &= \sum_{x=0}^n x \binom{n}{x} p^x (1-p)^{n-x} \\ &= \sum_{x=1}^n x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \\ &= np \sum_{x=1}^n \frac{(n-1)!}{(x-1)![(n-1)-(x-1)]!} p^{x-1} (1-p)^{(n-1)-(x-1)} \\ &= np \sum_{y=0}^{n-1} \binom{n-1}{y} p^y (1-p)^{n-1-y} \quad [y = x-1] \\ &= np \cdot 1 = np \end{aligned}$$

4. From p. 20, for a geometric random variable show

$$\sum_{x=1}^{\infty} P_X(x) = 1$$

Solution:

$$\begin{aligned}\sum_{x=1}^{\infty} P_X(x) &= \sum_{x=1}^{\infty} (1-p)^{x-1} p \\ &= p \sum_{x=0}^{\infty} (1-p)^x \\ &= p \left[\frac{1}{1-(1-p)} \right] = \frac{p}{p} = 1\end{aligned}$$

5. From p. 20, for a geometric random variable show

$$E(X) = \frac{1}{p}$$

Solution:

$$\begin{aligned}E(X) = \sum_{x=1}^{\infty} x P_X(x) &= \sum_{x=1}^{\infty} x (1-p)^{x-1} p \\ &= \frac{p}{1-p} \sum_{x=0}^{\infty} x (1-p)^x \\ &= \frac{p}{1-p} \left[\frac{1-p}{[1-(1-p)]^2} \right] = \frac{1}{p}\end{aligned}$$

6. From p. 21, for a Poisson random variable show

$$\sum_{x=0}^{\infty} P_X(x) = 1$$

Solution:

$$\begin{aligned}\sum_{x=0}^{\infty} P_X(x) &= \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} e^{-\lambda} \\ &= e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} \\ &= e^{-\lambda} e^{\lambda} = 1\end{aligned}$$

7. From p. 21, for a Poisson random variable show

$$E(X) = \lambda$$

Solution:

$$\begin{aligned} E(X) &= \sum_{x=0}^{\infty} x P_X(x) = \sum_{x=0}^{\infty} \frac{x \lambda^x}{x!} e^{-\lambda} \\ &= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} \\ &= \lambda e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} \\ &= \lambda e^{-\lambda} e^{\lambda} = \lambda \end{aligned}$$

8. From p. 26, for an exponential random variable show

$$\int_0^{\infty} f_X(x) dx = 1$$

Solution:

$$\begin{aligned} \int_0^{\infty} f_X(x) dx &= \int_0^{\infty} \lambda e^{-\lambda x} dx \\ &= \left[-e^{-\lambda x} \right]_{x=0}^{\infty} \\ &= 0 - [-1] = 1 \end{aligned}$$

9. From p. 26, for an exponential random variable show

$$E(X) = \frac{1}{\lambda}$$

Solution:

$$\begin{aligned} E(X) &= \int_0^{\infty} x f_X(x) dx = \int_0^{\infty} \lambda x e^{-\lambda x} dx \\ &= \left[-x e^{-\lambda x} \right]_{x=0}^{\infty} - \int_0^{\infty} -e^{-\lambda x} dx \\ &= 0 + 0 + \left[\frac{-1}{\lambda} e^{-\lambda x} \right]_{x=0}^{\infty} \\ &= 0 - \frac{-1}{\lambda} = \frac{1}{\lambda} \end{aligned}$$

Note that we used integration by parts with $u = x$ and $v = -e^{-\lambda x}$.

Step	Router 2 $N_2(1), D_2(1)$	Router 3 $N_3(1), D_3(1)$	Router 4 $N_4(1), D_4(1)$	Router 5 $N_5(1), D_5(1)$	Router 6 $N_6(1), D_6(1)$
Initial	1 2	2 3	1 3	4 6	3 5
lines(1,2) down	3 4				
(2- > 1,3,4)		2 5			
(3- > 2,5,6)	3 6				3 7
(2- > 1,3,4)		2 7			
(3- > 1,7)	4 6				5 7

Table 3: Shortest paths from each node to Node 1 after Link(1,2) goes down.